Numerical Integration

-As we've seen often there aren't near as many rules for integration as there are for differentiation.

-In fact, there are many functions that don't have a simple antiderivative and we need use an approximation method such as the limit-sum definition we originally used to define a definite integral.

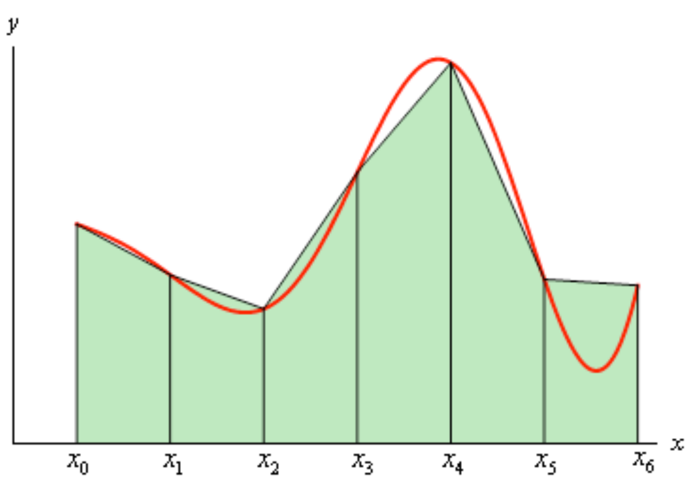
-One of the most popular of these is the Trapezoid Rule.

Trapezoid Rule

Just as we did with rectangles we will need to partition the interval into n-subintervals:

$$\Delta x = \frac{b-a}{n}$$

An example of this with 6 subintervals would be



The area of a trapezoid is given by:

$$A_i = \frac{\Delta x}{2}\left[f\left(x_{i-1}\right)+f\left(x_i\right)\right]$$

If we remember and use the definite integral as the summation of multiple parts we get:

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left[f(x_0)+f(x_1)\right]+\frac{\Delta x}{2}\left[f(x_1)+f(x_2)\right]+\cdots+\frac{\Delta x}{2}\left[f(x_{n-1})+f(x_n)\right]$$

If we clean this up and generalize this we get the **<u>Trapezoidal Rule</u>**:

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left[f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_{n-1})+f(x_n)\right]$$

Notice that the coefficients of the terms follow the pattern 1, 2, … ,2, 1.

<u>Example</u>

Use 4 sub-intervals to approximate the value of $\int_0^2 e^{x^2}\,dx$.

First, find the width of the sub-interval:

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$$

This breaks the interval into the sub-intervals:

$$[0,0.5],[0.5,1],[1,1.5],[1.5,2]$$

Using the Trapezoidal Rule:

$$\int_0^2 e^{x^2}\,dx = \frac{1/2}{2}\left[e^{(0)^2}+2e^{(0.5)^2}+2e^{(1)^2}+2e^{(1.5)^2}+e^{(2)^2}\right]$$

$$\approx 20.64455905$$

If we compare this to the actual value we find that it is close but not perfect.

$$\int_0^2 e^{x^2}\,dx = 16.45262776$$